Research on Aircraft/Vortex-Wake Interactions to Determine Acceptable Level of Wake Intensity

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Nomenclature

b = wingspan
c = wing chord
C_L = lift coefficient = L/qS
C_l = rolling moment coefficient = M/qSb
C_{l_v} = wake-vortex-induced rolling-moment coefficient
C_{l_δ} = aileron-induced rolling-moment coefficient
L = lift
M = rolling moment
p = rolling velocity
$\dot{p}$ = roll acceleration
$\dot{P}$ = ratio of roll accelerations = $\dot{p}_v/\dot{p}_{\delta m}$ = $C_{l_v}/C_{l_{\delta m}}$
q = dynamic pressure = $\rho U_\infty^2/2$
$\dot{q},\dot{r}$ = pitch and yaw acceleration, respectively
S = wing planform area
U_∞ = velocity of aircraft
W = weight
x = distance in flight direction
X,Y,Z = $x/b, y/b, z/b$
y,z = distance in spanwise and vertical directions, respectively
γ = normalized vortex strength, Γ/bU_∞
Γ = vortex strength
δ = aileron deflection
δm = maximum aileron deflection
θ = pitch angle relative to vortex axis
ρ = air density
ϕ = roll angle
ψ = heading angle relative to vortex axis

Subscripts

f = following or probe aircraft
g = wake-generating aircraft
m = maximum
v = vortex

Introduction

THE capacity of an airport to accommodate aircraft depends on such factors as the number of runways, the landing and takeoff distances of the arriving and departing aircraft, the hourly distribution of arrivals and departures, and the spacing required between aircraft to insure that safety is not compromised on the ground or in the air. For some time, it has been recognized that the one factor that now dominates the minimum allowable spacing between aircraft is the hazard caused by the lift-generated vortices shed by aircraft. That is, the vortex wakes of aircraft persist long enough to force following aircraft to delay their arrival until the vortex wakes shed by previous aircraft have either descended below or been blown out of the flight corridor or have decayed to harmless levels. For these reasons, current minimum separation distances (Table 1) have been set for circumstances in which instrument flight rules (IFR) apply. The minimum distances listed in Table 1 for the various weight classes of transport aircraft are based primarily on observations of the lifetime and motion of wake vortices at airports. The distances listed indicate the amount of time needed for the vortices to decay to a harmless level in ground effect or to move downward and/or sideways so that they are well outside the flight corridor of any following aircraft. It is to be noted that larger separation distances are specified for small aircraft (including business types) following larger aircraft to insure a comparable margin of safety for the smaller aircraft, which are more susceptible to vortex encounters. The added distance allows more time for the vortices to move farther out of, and away from, the flight corridor and to decay. The separation guidelines must reflect worst-case conditions and so are often overly conservative. Since vortex descent and lateral motion depend on such variables as the wind, atmospheric turbulence, and proximity to the ground, transit velocity and dispersion of the vortices vary continuously with time. Since the wake velocities may not have decayed (e.g., when out of ground effect) and therefore could still be strong enough to be hazardous at and beyond the guideline distances specified in Table 1, the purpose of the separation guidelines is to prevent an encounter with the intense parts of the wake. In brief, the guidelines simply indicate when the approach corridor is safe for use by a following aircraft. That is, the vortices have either decayed sufficiently for safe passage or they have moved below and/or far enough away from the flight path to be harmless.

Based on factors other than the wake-vortex hazard, the minimum safe separation distance between aircraft in the airport environment and especially during landing and takeoff is estimated to be about 2 n.mi. Reduction of the presently specified spacings, which are listed in Table 1, to a uniform spacing of 2 n.mi. would simplify the landing and takeoff procedures and increase the runway use so that the capacity of

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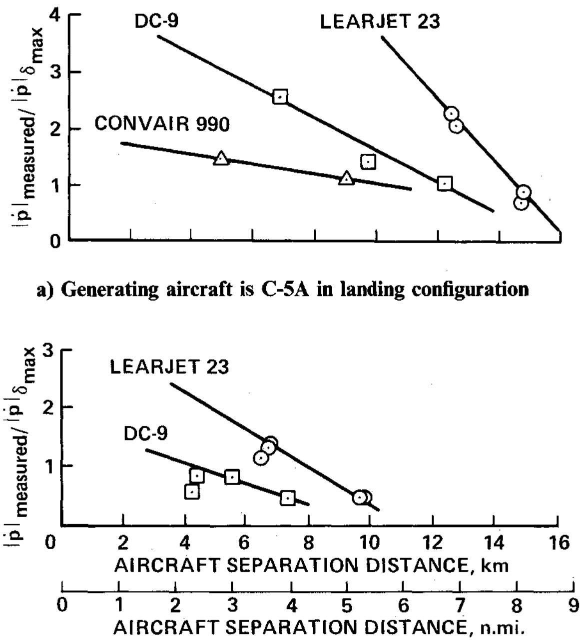


a) Generating aircraft is C-5A in landing configuration

b) Generating aircraft is Convair 990 in landing configuration

Fig. 1 Schematic of possible encounters with a lift-generated wake by a following aircraft.

Table 1 Current minimum separation distances, n.mi.

Following aircraft	Wake-generating aircraft		
	Small	Large	Heavy
Small	3	4	6
Large	3	3	5
Heavy	3	3	3

Note: Aircraft sizes defined by the weight ranges: small: $0 < W < 12{,}500$ lb; large: $12{,}500 \text{ lb} < W < 300{,}000$ lb; heavy: $W > 300{,}000$ lb.

airports would be increased to near their maximum value. If such a reduction were put into practice, the likelihood of a vortex-wake encounter increases substantially. In fact, it should be assumed that wake encounters will then occur so regularly that the wake-induced forces and moments must be manageable on a continuous basis by any penetrating aircraft. Since most aircraft cannot now safely enter the newly formed wakes of larger aircraft that have not decayed substantially, methods must be developed for reducing wake velocities to an acceptably safe level. That is, before such a reduction in separation distances can be made, the wake-vortex hazard must be reduced to a tolerable level at a distance of 2 n.mi. Since approximately the same flight path is used by the aircraft during landing and takeoff, the wake penetration is nearly aligned with the vortex axes rather than with a cross path encounter (Fig. 1). Hence, the roll-induced interaction is the one of most concern, and roll control is therefore the most important capability of the penetrating aircraft in countering the effects of the vortex wake. When an encounter occurs, both the generating and the following aircraft will probably be configured in their landing or takeoff modes, in which their flaps and landing gear are extended. Furthermore, since the penetrating aircraft is either landing or taking off, the nearness of the ground will limit the acceptable unexpected coherent motion caused by a vortex encounter.

The objective of the study reported here is to determine the amount by which the wake velocities must be decreased before they can be considered safe and tolerable for following aircraft within the restraints imposed by the airport environment. The study begins by examining the literature[1–35] to find out how much smaller the vortex-induced forces must be than the control capability onboard current transport aircraft if vortex encounters are to be safely tolerated on a regular basis. That is, the onboard control capability should not only be able to overcome the wake-vortex-induced forces but should also have a reserve for aircraft maneuvers, flight-path corrections, etc. This means that the vortex-induced forces should be smaller than the maximum available control forces by some factor that will be established on the basis of encounter information contained in the literature. When such a factor is established, the maximum torque permitted by a vortex wake can be used to determine the characteristics of an adequately alleviated wake. Knowledge of the extent to which the intensity of vortex wakes must be reduced provides researchers with a goal in their wake-alleviation efforts. Since the torque loading by the vortex wake needs to be calculated for various aspects of these investigations, a survey[36–45] of applicable techniques is presented in the Appendix. Two of the several methods discussed in the Appendix are then used with several typical vortex wakes[46–54] to study the effect of the structure of the vortex wake on torque in order to assess the magnitude of the alleviation problem and to study the changes in wake-induced forces as various parameters are changed. Finally, alleviation concepts[55–76] are discussed to indicate possible research directions that might yield the wake alleviation needed if the aircraft spacing is to be reduced to the desired 2-n.mi. separation.

Background Information

The literature[2–35] (presented in chronological order in the references) on the interaction of aircraft with wake vortices is first reviewed in a survey of information on the level of vortex-induced loads acceptable to encountering aircraft. One of the first papers written about the possible hazard posed by the wakes of large aircraft appeared when the DC-6 was put into service in about 1950. Bleviss[2] estimated the decay rates of the various wake components and concluded that the so-called propeller wash spread quickly enough that it could be considered nonhazardous to other aircraft flying several spans behind the generating aircraft. He noted, however, that the lift-generated vortices in the wake decay so slowly that they persist for some distance behind the generating wing. In 1955, in order to better understand the structure and slow rate of decay of lift-generated vortices, Kraft[3] made measurements in flight of the wake velocities, with a P-51 ($W_g = 8800$ lb) as the wake generating aircraft and with two larger aircraft as the probe aircraft ($W_f = 13{,}480$ and 16,400 lb). In addition to obtaining some of the first in-flight measurements of vortex structure, Kraft found that the strength of the vortices did not decrease appreciably for about 35 s behind the generating aircraft and that the propeller wake is not detectable at distances in excess of about 1000 ft behind the aircraft. The apparently rather slow decay of the vortices, when compared with the data correlation of Iversen,[53] may be attributable to atmospheric structure present at the time of the flight test.[75] The across-wake penetration and data-reduction techniques developed for these experiments are similar to those used in other ground-based[50] and flight[28] investigations of wake vortices. When several axial penetrations were made into the wake, the pilot found that it was difficult to maintain a precise course and that the vortices caused disturbances similar to severe atmospheric turbulence. No mention was made of overpowering vortex-induced roll excursions, probably because the penetrating aircraft was about twice as large as the generating aircraft. Breakup of the wake occurred in about 60 s after the vortex lines became very sinuous from motions brought about by atmospheric eddies. It was also noted that the sinuous shape of the vortices made it difficult for the pilot to find and stay in the vortices for prolonged measurements.

Several investigations[4–7] were carried out from 1960 to 1963 to define the vortex-wake structure and to estimate the loads imposed on a penetrating aircraft as it enters the wake from different directions (Fig. 1). The results showed that the loads imposed by a wake may overpower the control capability and may exceed the structural limits of the penetrating aircraft. Based on flight test results and on estimates of vortex decay by use of a Lamb's vortex model,[6,9] the time required for the wake to decay to a safe level was estimated to be in excess of 3 min (over 6 n.mi. at landing velocity).

In order to explore separation requirements more fully, flight tests were conducted with various size combinations of transport aircraft.[8–12,18,20,22] In order to be certain that the most intense parts of the wake were penetrated, the vortices were made visible by injection of dust, smoke, or oil mist into the wake. The injected particulates identify the vortex cores by collecting in smoothly shaped tubes around the centers of the vortices. Penetration of the cores defined in this way by the probe aircraft causes a combination of yaw, pitch, and roll as brought about by the aerodynamic forces on the vertical and horizontal tail surfaces and on the wing by the rotary velocities of the vortex wake. The resulting acceleration and motion of the following aircraft must be quickly offset with control inputs by the pilot, or the excursions will become large enough to be possibly hazardous. The descriptions presented by the pilots[9,11,12,20,22] provide interesting insight into the complex interaction of the aircraft with a vortex wake as the pilots tried to hold the aircraft in an acceptable orientation. Although all six components of motion are experienced in an encounter, it was observed that the roll acceleration and maximum bank angle were the parameters most affected by an axial penetration of a vortex wake. In fact, a small jet aircraft (Lear Jet-23) rolled 360 deg when it encountered[9] the center of a vortex in the wake of a C-5A. Furthermore, roll excursions of 40–80 deg were not uncommon with other aircraft. Even with aircraft of nearly the same size, it is necessary for the pilot to apply control motions quickly to offset the wake-induced motions, or else the excursions will become large enough to be considered hazardous.

One of the main objectives of the flight tests was to determine a minimum distance for aircraft of various sizes at which it is safe to enter the wakes of other aircraft. The criterion used was the ratio of roll acceleration due to the vortex-induced rolling moment to the roll acceleration brought about by the maximum roll control by aileron deflection of the probe aircraft; i.e., this ratio is defined as

$$\dot{P} = \dot{p}_v / \dot{p}_{\delta m}$$

or, in terms of rolling moment coefficients,

$$\dot{P} = C_{l_v} / C_{l_{\delta m}}$$

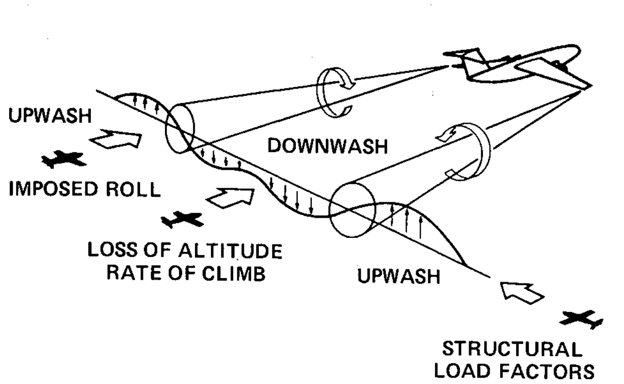


Fig. 2 Ratio of vortex-induced roll acceleration to maximum lateral control power as a function of separation distance at an indicated airspeed of 150 knots (from Ref. 11).

When the roll-control ratio $\dot{P}$ is equal to one, the probe aircraft is just able to hold its own against the vortex-induced rolling moment if the controls are applied instantaneously as needed. Since the flight path does not usually line up with the vortex axis for large distances, and since the aircraft also is often thrown out of the vortex wake by the vortices, the forces and moments are usually temporary or intermittent, thereby causing scatter in the data. Therefore, the upper bound of the roll-control ratio, as measured in flight at various distances, indicates a minimum distance behind the generating aircraft at which the following aircraft can just maintain controllability. It was found that when the parameter $\dot{P}$ was less than one, the inertia of the probe aircraft and the impulsive character of the vortex-induced force field tended to limit the excursions to a level that seemed safe under visual flight conditions at an altitude of around 10,000 ft. Therefore, a graph of the roll-control ratio, as measured in flight at various distances behind two different wake-generating aircraft (Fig. 2, reproduced from Ref. 11), provides a means for finding a so-called safe minimum aircraft spacing for penetrating a given vortex wake. Similar results for other combinations of aircraft sizes were used, along with other available information, to determine separation guidelines at airports. Hence, at the time of these tests, the answer to the question being treated in this paper was that the vortex-induced rolling moment should be less than the roll control available onboard any following aircraft. It was recognized, however, that although the roll excursions experienced at altitude are perceived to be nonhazardous, they would probably be unacceptable near the ground during the last part of a landing.

In an effort to include all three components of the accelerations due to the wake-induced moments, a more complete encounter parameter, a spin-control parameter, was considered by Jacobsen and Short.[28]

$$\dot{\Omega} = [(\dot{p}_v/\dot{p}_{\delta m})^2 + (\dot{q}_v/\dot{q}_{\delta m})^2 + (\dot{r}_v/\dot{r}_{\delta m})^2]^{\frac{1}{2}}$$

It was found, however, that the data correlated just as well as if only the roll-control portion $\dot{P}$ of the parameter $\dot{\Omega}$ were used. Although this conclusion was taken as apparent by others, it is comforting to know that a comparison was made and that the parameter $\dot{P}$ was confirmed as adequate. Since the data confirmed that the axial penetration of vortices is dominated by rolling motion, the more complicated parameter never came into common use. As a result, the discussion here considers only the roll-control ratio $\dot{P}$; i.e., the ratio of the roll acceleration resulting from the vortex to the roll acceleration brought about by full deflection of the ailerons.

Another objective of the flight tests was to determine the controllability of the probe aircraft in various vortex wakes as a function of the separation distance. Scatter in the data, and the difficulty in penetrating and holding the follower aircraft in or near the vortex core, made it difficult to form well-defined decay rates. Later, it was shown by Greene[75] that the stability of, and turbulence in, the atmosphere are responsible for some of the wide variation in the flight test results. The flight test results did show that large flight-path distortions could on occasion be caused by an encounter with wake vortices at distances of about 10 n.mi. and that the vortices decay rather slowly unless some sort of instability sets in. If the following aircraft is the same size or larger than the wake-generating aircraft, the interaction is manageable by the control forces onboard the following aircraft. Some of the manageability is attributed to the fact that the penetrating aircraft either passes through the vortex (when it has a sinuous shape) or is thrown out of the vortex so that the time-averaged impulse imparted by the wake on the aircraft remains within controllable limits. The dynamics of the encounter[16,24,25] showed that the velocity field of the vortices often causes the penetrating aircraft to be deflected away from the center, or most hazardous part, of the wake, which tends to reduce the total roll excursion of the penetrating aircraft. Conversely, it was also found that control

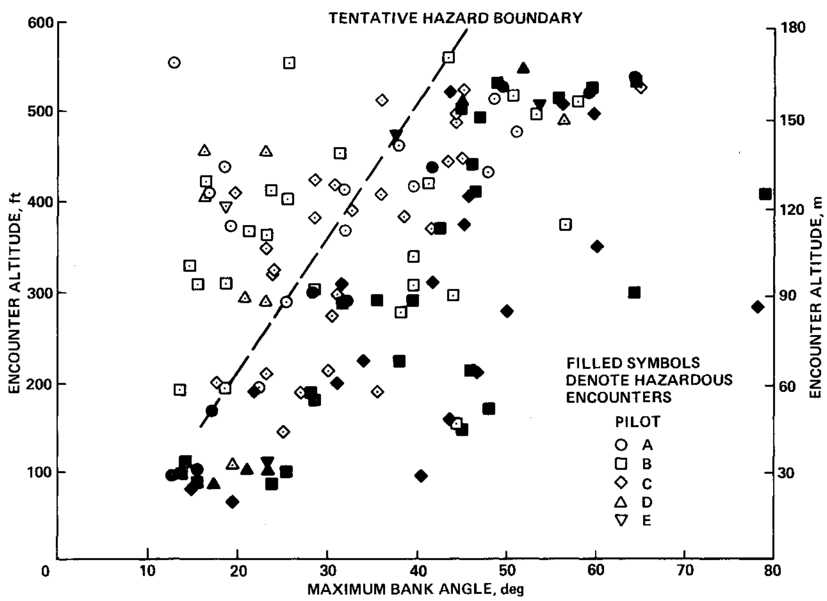


Fig. 3 Display of data for maximum bank angle from simulated encounters for all entry conditions tested under VFR conditions (from Ref. 21).

efforts on the part of the pilot could inadvertently guide the probe aircraft into a more hazardous part of the wake. Narrative descriptions[3,8–12,16–20] of the encounter process, both as experienced in flight and as predicted by numerical analysis, indicate a wide variation in possible motions that can be induced on the following aircraft by a vortex wake.

The slow rate of vortex decay and dispersion in the flight tests was also confirmed in ground-based experiments carried out in wind tunnels and water tow tanks. The tests were directed at gaining an understanding of the structure, decay rates, and instabilities of lift-generated wakes. It was found by Ciffone[49] and Ciffone and Orloff[51] that the vortices may change very little for about 40 spans behind the generating wing. The vortex structure was then found to be approximated quite closely by simple inviscid theory.[18,47,48,52] Downstream of this so-called plateau region wherein very little decay occurs, the vortices decay or disperse as $t^{1/2}$ or as the inverse of the square root of the distance behind the generating aircraft. Since a wide variety of wing planforms exhibited the same plateau and decay characteristics, Iversen[53] correlated the data obtained in both ground-based facilities and in flight into a single curve. These results emphasized that naturally occurring vortex decay would not provide enough reduction in wake-induced rolling moments to influence greatly the separation distance deemed safe. In fact, 40 spans behind a B-747 is about 1.5 n.mi. and, at that distance, decay is estimated to be only beginning. Therefore, a significant change in the wake at 2 n.mi. is not to be expected unless some sort of radical change is made in the structure of lift-generated wakes to enhance decay or dispersion.

Although it had been realized for some time that the infrequent occurrence of vortex encounters during landing was attributable to the downward and outward motions of the vortices, ground-based instrumentation installed at airports began to provide firm data[62,63,67,71] on vortex positions as a function of time behind the generating aircraft. It was found that not only do the vortices move out of the flight corridor used by aircraft to land and takeoff but the nearness of the ground makes the vortices decay much faster than when they are isolated at altitude. Numerical simulations[14,17,24,26] of the vortex trajectories, of the corresponding hazardous regions, and of aircraft penetrations predict the same kinds of behavior. These results provided zones or regions around the flight path to be avoided by following aircraft and supported the measurements made of aircraft vortices during landing at airports. The numerical studies also indicated an absence, or very infrequent occurrence, of wake-vortex encounters by aircraft at airports. That is, the low rate of vortex encounters at airports is attributable to the fact that the vortices have moved out of and away from the flight path of landing aircraft to produce what might be called a naturally occurring wake-avoidance system. The purposeful encounter of vortex wakes at about 10,000-ft altitude was used to determine their severity as a function of downstream distances; this was then replaced by studies to determine how long and under what circumstances wake vortices are far enough from the flight paths so as not to constitute a hazard.

Since the current separation guidelines allow enough time for the wake vortices to move far enough from the flight corridor, and/or decay, during landing and takeoff, the rotary flowfields of the vortices are too far from the flight path of any following aircraft to affect its motion. If, however, the separation guidelines are reduced to a uniform value of 2 n.mi., it is much more likely that encounters with the stronger parts of the vortices will take place more often. A vortex encounter must, therefore, be so benign that the following aircraft remains easily controllable along its intended landing corridor. An ideal wake-alleviation scheme would reduce the vortex-induced motions to such a low level that the pilot could not distinguish them from atmospheric turbulence. Reduction of the wake intensity to such a low level is no doubt unnecessary. The more practical and feasible solution is to reduce the wake intensity to a level at which excursions are just tolerable to pilots when vortex encounters occur on a regular basis.

In order to obtain information on the levels of acceptable vortex-induced motions, tests were conducted with piloted ground-based simulators[21,23,25,31] to obtain repeatable data on excursions caused by wake-vortex encounters. The tests included not only the vortex encounters but also atmospheric turbulence, along with the usual piloting duties associated with the airport environment. The forces and moments on the probe aircraft were computed at each increment of time by use of a strip-theory type of analysis that was fast enough to provide data on a real-time basis for the six degrees of freedom of motion. The piloting task was to fly a 3-deg glide slope toward a landing with an abort capability if desired. The pilots who had had flight experience with wake-vortex encounters reported[21,23] that the simulations were quite realistic and a good representation. Also, the simulations were judged to be a useful and valid method for establishing hazard criteria. After a number of simulated encounters had been flown under both visual flight conditions (VFR) and instrument flight conditions

(IFR), the separation of occurrences into hazardous and nonhazardous categories correlated best with maximum roll or bank angle.[21,23,25] The data used to infer boundaries between hazardous and nonhazardous conditions are reproduced[23] in Fig. 3 for a range of altitudes and bank angles. The boundaries are summarized in Fig. 4 for both IFR and VFR. It was concluded from the simulations that, under IFR conditions, a maximum roll angle of more than 7 deg is perceived as hazardous at altitudes of 200 ft or less. The primary reason given by the pilots for rating an encounter as hazardous was proximity to the ground and subsequent altitude loss caused by the encounter.

A similarly well-defined boundary between hazardous and nonhazardous conditions was not found for either roll rate or roll acceleration. It may be that since the vortex intensity was for larger aircraft [$\Gamma = 92.9$–$185.8\ m^2/s$ (1000–2000 ft^2/s)], the

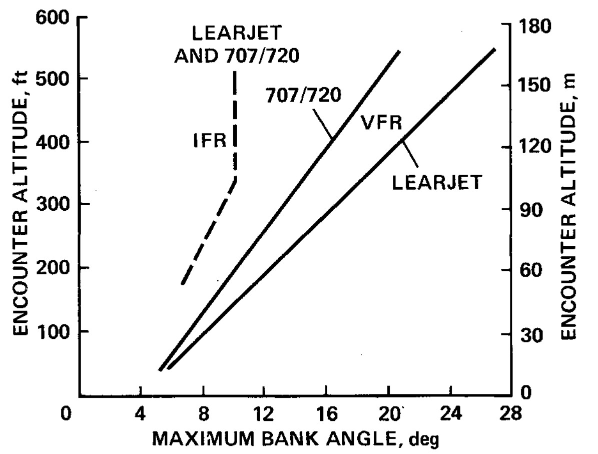
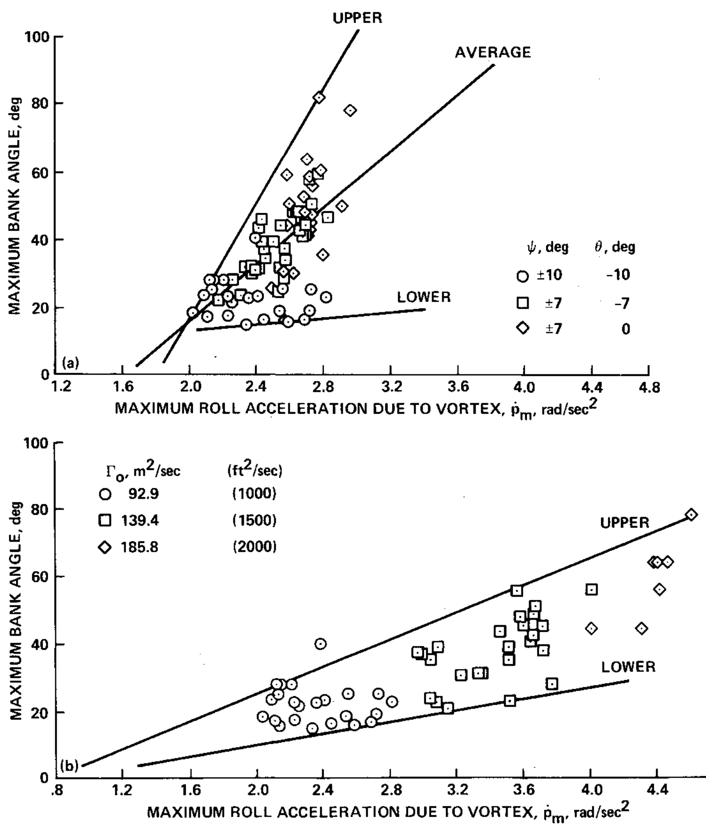


Fig. 4 Summary of data from simulated encounters indicating the boundaries between hazardous and nonhazardous conditions for all entry conditions tested with a Lear Jet and a B-707/720 aircraft (from Ref. 23).

roll velocities and accelerations that occurred in the simulations did not go to low enough values to define a boundary completely. In the absence of enough data at the lower values of roll acceleration to define a sharp boundary between hazardous and nonhazardous roll accelerations, the available encounters are extrapolated to the low altitudes of interest. When such an extrapolation is made (Fig. 5), the maximum tolerable roll acceleration is estimated to be about 1.6 rad/s^2, which exceeds the value of 1.15-rad/s^2 capability of the Lear Jet used in the simulation. Since the acceptable value exceeds the capability of the aircraft, either the extrapolation in Fig. 5 is invalid or roll acceleration is an insensitive indicator of hazard.

Two more values can be inferred from the data of Sammonds and Stinnett[21] when they divide their roll acceleration data (reproduced in Fig. 6) into two groups. In the first (Fig. 6a), the data for a given vortex strength are plotted as a function of maximum bank angle while the entry angle into the vortex is varied. In the second (Fig. 6b), the entry angle into the vortex is held constant while the vortex strength is varied. Even though some sort of relationship appears to exist in the data, extrapolation to low values of bank angle can be only approximated. Therefore, lines drawn through the center of the data points, or along the upper or lower edges of the data scatter, provide an estimate of the roll acceleration not to be exceeded if the encountering aircraft is to roll no more than 5 deg or so. If such a connection and the extrapolation are valid, the values estimated for maximum acceptable roll acceleration due to a vortex encounter appear to range from about 1.6 rad/s^2 to about 0.8 rad/s^2. In order to be conservative, the 0.8-rad/s^2 value is chosen as representative of maximum tolerable values for $\dot{p}_v$. A value for the roll-control ratio $\dot{P}$ can be calculated for aircraft by using the appropriate value for $\dot{p}_{\delta m}$. From Tinling,[29] typical values (Table 2) for the roll-control acceleration range from about 0.6 rad/s^2 for large aircraft to 1.8 rad/s^2 for small aircraft. Since the Lear Jet characteristics were used for the aircraft used in the ground-based simulations, the appropriate value (Table 2) for $\dot{p}_{\delta m}$ is 1.15 rad/s^2. The corresponding value for the roll-control ratio $\dot{P}$ is then (0.8/1.15), or about 0.7 for the Lear Jet.



a) Data for constant vortex strength [$\Gamma = 92.9\ m^2/s$ (1000 ft^2/s)], variable entry angle into vortex

b) Data for constant encounter angle ($\psi = \pm 10$ deg, $\theta = -10$ deg), variable vortex strength

Fig. 5 Display of data for maximum roll acceleration from simulated encounters for all entry conditions tested under VFR conditions (from Ref. 21).

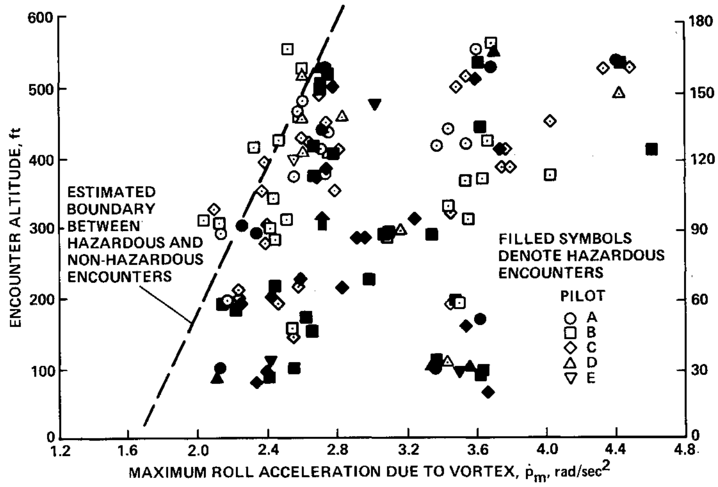


Fig. 6 Data from simulated encounters illustrating variation of maximum bank angle with maximum roll acceleration due to the vortex (from Ref. 21).

Table 2 Aircraft characteristics[12,27]

Aircraft	Wing span, ft	Available roll control, $C_{l\delta m}$	Roll-control power, $\dot{p}_{\delta m}$, rad/s²
Cessna 210	36.6	0.056	—
T-37B	33.8	0.060	—
Lear Jet	34.0	0.047	1.15
PA-30	36.0	—	1.87
DC-9	89.4	0.067	0.97
B-737	93.0	0.097	—
B-727	108.0	0.092	0.62
Convair 990	118.0	0.059	—
B-707	145.8	0.080	—
B-747	195.7	0.068	—

A value for $\dot{P}$ between 0.5 and 1.0 appears to be of about the correct magnitude because Smith[19] reported that pilots of the T-37B probe placed the safe separation distance behind a B-747 alleviated configuration at about 5.5 n.mi. At that distance, the maximum measured rolling moment was about 0.6 of the aileron capability of the T-37B probe. A similar figure for the nonalleviated wake of the B-747 is a bit uncertain because the data do not go beyond the 13-n.mi. distance judged safe by the pilot. However, data at the 10–11n.mi. distance scatters from about $\dot{P} = 0.4$–0.8. Since these flight tests were all VFR and at altitude, it is not clear how they are related to the more delicate situation that exists with IFR conditions at low altitude. The foregoing results are in agreement with the value of 0.5 estimated for $\dot{P}$ by Burnham[32] while carrying out a study of wake-alleviation requirements.

The finding that a vortex encounter is considered nonhazardous if the maximum roll excursion is below a certain value prompted studies of feasibility[13,15] and effectiveness[29] of an automatic control system on maximum roll angle. It was observed that since the simulator experiments were designed to make the vortex encounter unexpected, the pilot response during a typical encounter first consisted of a time delay[29] of about 0.4 s. During this time delay, the aircraft undergoes the initial accelerations by the velocity field of the vortex. The pilot then applies roll control in proportion to the perceived roll rate. It was then reasoned that a considerable reduction in roll excursion could be achieved if an automatic system was used to command immediate action. The numerical analysis then carried out by Tinling[29] showed that when the full amount of roll control is used with an automatic system and, when the parameter $\dot{P}$ is less than one, the angle of roll can be kept within acceptable limits. An illustration of the effectiveness of the automatic control system is presented in Fig. 7 (reproduced from Ref. 29) for a range of roll accelerations. It is apparent then that the onboard capability of the following aircraft can increase the value of the roll-control ratio $\dot{P}$, which is required to qualify a wake as nonhazardous or acceptably alleviated. If the onboard response is minimal, the value of $\dot{P}$ that is required could be as low as the 0.4 value estimated on one of the flight tests.[19] If the onboard capability is maximized, the acceptable value for $\dot{P}$ could approach 1.0.

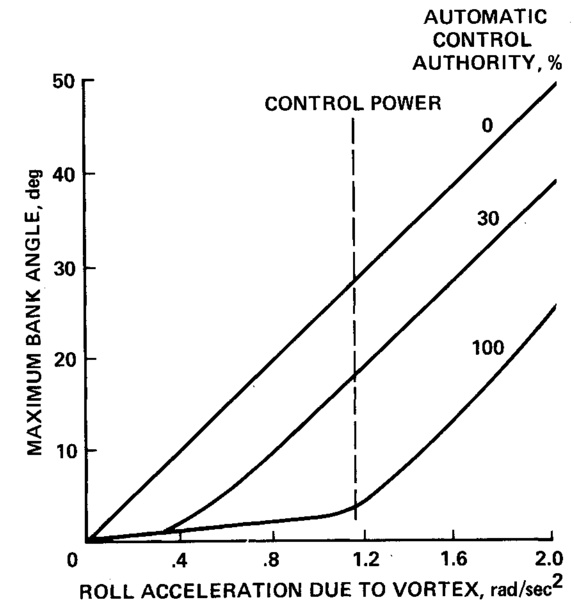


Fig. 7 Estimate of maximum bank angle due to vortex encounters for several levels of roll-control authority for automatic system based on numerical analysis (from Ref. 29).

In summary, the foregoing estimates for $\dot{P}$, which include the response time of the pilots, range from 0.4 to 0.8. Hence, a single approximate value for $\dot{P}$ that could be used as a general rule of thumb for all aircraft is $\dot{P} \leq 0.5$. When this value is combined with typical values for the maximum aileron control capability contained in Table 2, it appears that the wake-vortex-induced rolling-moment coefficients should not exceed about $C_{l_v} = 0.03$. This value is close to the value 0.033 ($= 0.7 \times 0.047$) calculated for the Lear Jet. Consideration is next given to wake-alleviation concepts in order to ascertain the amount of reduction in wake intensity required before separation distances can be decreased to the desired 2-n.mi. spacing.

Possibilities for Wake-Vortex Accommodation

Alternatives

Consideration is first given to the factors that affect the intensity of wake-vortex encounters and to how various parameters affect the magnitude of the wake-induced excursions. The literature on wake-vortex alleviation[55–76] is then reviewed to examine some current possibilities for achieving the reduction in wake-vortex intensity needed to satisfy the requirement that $C_{l_v} \leq 0.03$ for all following aircraft. The magnitude of the problem is illustrated by recalling[60,61] that the wake vortices trailing from a B-747 induce a rolling moment on a Lear Jet type of following aircraft about equal to $C_{l_v} = 0.12$ at 1–2 n.mi. behind the generating aircraft. Hence, an accommodation of some sort by a factor of about 4 must be made. The possibilities for enabling the following aircraft to endure vortex wakes at 2-n.mi separations include: 1) wake avoidance, 2) increasing the control power of all aircraft to the point at which they can maintain an acceptable attitude throughout the wakes generated by any aircraft that may possibly land or take off ahead of them, 3) decreasing the intensity of the velocities in the wakes of the generating aircraft so that the current control capabilities are sufficient (i.e., alleviate the vortex wakes), or 4) some combination of these techniques.

Each of these possibilities has its own set of difficulties. For example, wake avoidance at the 2-n.mi. separation requires that following aircraft not use the same flight corridor because the decay rate and the natural motions of the vortices to move out of a single flight corridor are both too slow to provide safe passage on a consistent basis. Hence, avoidance would require that aircraft use a series of corridors spaced along the runways so that enough distance is provided between flight paths to reduce vortex encounter probabilities to a negligible level. Such an avoidance system reduces the latitude available for variations in touchdown point and would no doubt require longer runways to retain the present margin of safety.

The second possibility mentioned is to increase the roll-control power onboard the penetrating aircraft to enable control to be maintained throughout any vortex wake encountered during landing. Such a change is probably impractical because it would require large changes in the characteristics of the aircraft. Since transport aircraft are designed for efficiency in cruise, they cannot be readily adapted to special requirements (which may degrade cruise performance) to cope with occasional situations during landing and takeoff. For example, an increase in the roll-control power of an aircraft by a factor of 4 would require major changes. Since the aileron rolling-moment capability typical of current transport aircraft is about $C_{l_{\delta m}} = 0.06$, an upgrade to over 0.2 would be a major undertaking and would probably be found to be an impractical solution. That is, large changes are required not only in the wing structure but also in the mechanisms needed to apply such large roll-control forces rapidly when a vortex is encountered and to retain normal roll control for other flight purposes. The combination of large control surfaces and rapid deflection for a variety of flight circumstances poses difficult design problems for a large fleet of transport aircraft and does not appear to be a satisfactory approach to a solution.

The most attractive solution in the foregoing list is to alleviate the rotary velocities in the lift-generated wakes to the degree where the rolling-moment coefficients induced on any following aircraft will be less than $C_{l_v} = 0.03$. As mentioned previously, this requires roughly a fourfold reduction in the rotary velocities of the wake. It can be expected that such a reduction is required for all large aircraft. Furthermore, any penalties imposed by the alleviation scheme and the costs of implementation should be small in comparison with the savings generated by higher airport capacities. The first alleviation scheme[60–62] to demonstrate a considerable reduction in wake intensity used a special span-load variation to trigger wake instability. Unfortunately, the technique could not be implemented because the alleviation became ineffective when the landing gear was deployed. Two other schemes have also been demonstrated as capable of providing the desired degree of alleviation, but the feasibility of their implementation is in doubt. The first[66–68] of these two employs small vertical surfaces on the upper surface of the wing to inject extra vortices into the wake to disperse the vorticity. The second, found inadvertently during flight tests that used spoilers to achieve wake alleviation[71] on the B-747, utilizes a vortex instability[74] to bring a chaotic character to the wake. On penetration of the wake of a B-747 at 1.5 n.mi., the pilots commented[71] that no coherent rotary motion was perceived by the probe aircraft and that flight through the wake resembled a ride over a bumpy road.

Even though these three alleviation concepts have been found to reduce wake-vortex hazards significantly, guidelines for the design of wings or aircraft that shed acceptable wake vortices have not been developed. It is concluded, therefore, that wake alleviation is the most promising approach for achieving acceptably safe conditions for all aircraft at the 2-n.mi. separation distance. This has been the goal of the wake-alleviation program being conducted by various government organizations.[10,61,63,67] Some of the objectives of the program are to provide design guidelines that will produce wakes in which the rotary velocities are small enough to impose only a small rolling moment on penetrating aircraft. The velocity gradients must also be so small that an abrupt entry into or departure from a lift-generated wake will result in neither aircraft excursions nor accelerations that go beyond the limits determined as safe and acceptable. Furthermore, the vortex-wake structure must be so benign that the magnitude of the loads is nearly independent of the direction of penetration or of the amount of curvature of the wake vortices and of any intermittency or changes in the vortex structure along its axis; e.g., those brought about by instabilities or breakdown of the wake or by atmospheric effects. That is, the penetrating aircraft must be able to enter and remain at the most intense part of the vortex wake on a continuous basis without excessive loads being induced on any components of the probe aircraft.

Predictions of Rolling Moment

The study of the dynamics of vortex wakes and the interaction of aircraft with them requires an ability to estimate the aerodynamic loads on the surfaces of the probe aircraft when the wake structure is known. Although all of the components of forces and of moments can be calculated, the one of most interest for the present study is the rolling moment, or torque, imposed by the wake on the penetrating aircraft. As described in the Appendix, the methods available range from quite simple strip-theory-type analyses to fairly involved finite-difference solution methods. As recommended in the Appendix, the two methods used here consist of a modified strip theory that incorporates features suggested by Munk,[36] Jones,[38] and Maskew[41] and a vortex-lattice method developed by Hough.[42] Both methods assume that the wake is not changing with time and that the penetrating wing is fixed at some point in the wake so that the calculation can be treated as a steady-state computation.

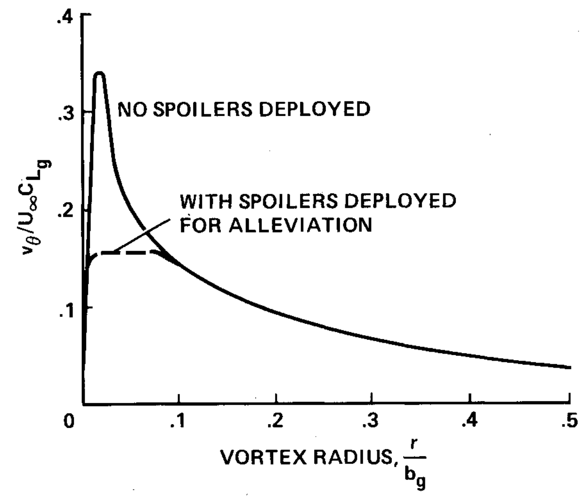
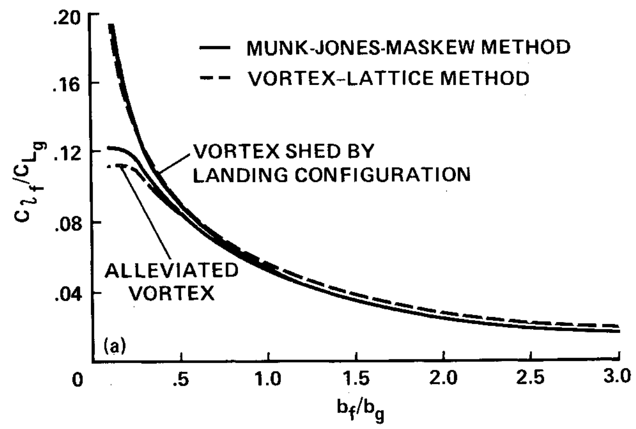
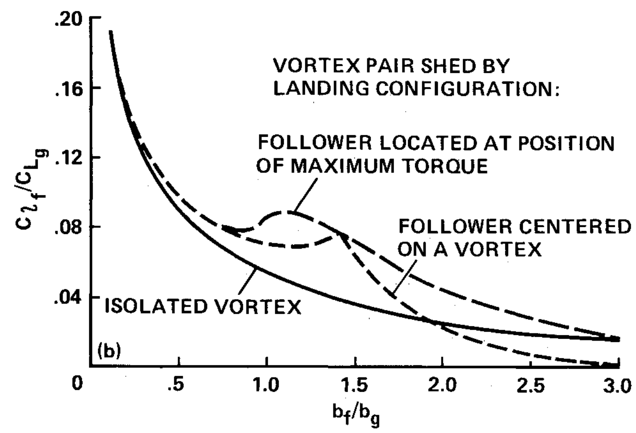


Fig. 8 Rolling-moment coefficient as a function of span ratio as induced on a wing of rectangular planform by various wake configurations.

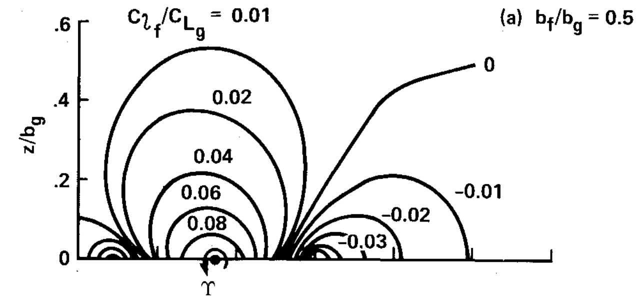


a) Comparison of two rolling-moment prediction methods for isolated vortices with alleviated and nonalleviated structures

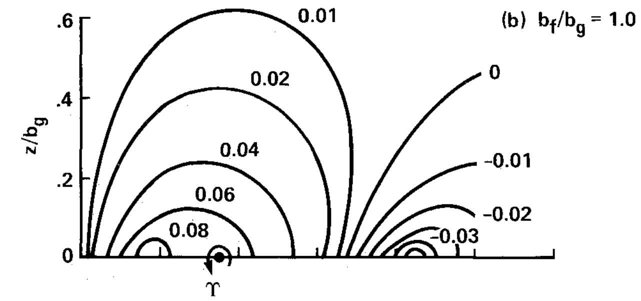


b) Comparison of wake-induced rolling-moment coefficient for three wake situations

Fig. 9 Velocity profiles assumed for structure of vortices shed by typical transport aircraft in landing configuration.[50,62,72]



[a]Initial configuration of wake vortices as marked by oil mist smoke generators.



[b]About one mile behind aircraft.

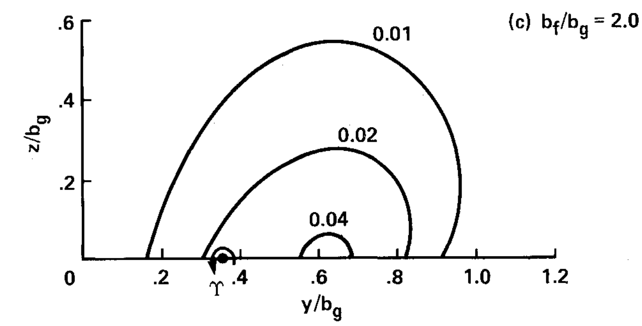


Fig. 10 Contours of equal rolling moment coefficient induced on a following wing of rectangular planform by a vortex pair shed by large transport aircraft in its landing configuration.

These two methods are first applied to explore the torque or rolling-moment imposed on a wing that is embedded in a flowfield composed of two different vortex structures[50,62,72] (Fig. 8). The first vortex-wake structure is representative of large aircraft in their landing configuration; the second is representative of the same type of configuration with the addition of turbulence injection to reduce the rotational velocity in the core regions of the vortex pair.[72] In order to illustrate the variations to be expected as the span ratio changes, several examples are presented in Figs. 9–11. Figure 9 begins by presenting the rolling-moment coefficient induced on a following wing of rectangular planform by alleviated and nonalleviated vortex structures. The predictions are presented as a function of the ratio b_f/b_g of the wingspan of the penetrating or following wing to that of the generating wing. In Fig. 9a, the penetrating wing is assumed to be located at the center of an isolated vortex. It is to be noted that the reduction of the rotary velocities in the vicinity of the vortex center by turbulence injection (Fig. 8) modifies the rolling moment only for the smaller following wings. This result illustrates the need for modification of the entire structure of the vortex wake if effective wake alleviation is to be accomplished. As noted previously,[50] the strip theory that combines suggestions by Munk, Jones, and Maskew is in very good agreement with the vortex-lattice method.[42] The differences between the two predictions are small compared with the uncertainties usually experienced in obtaining test data.[50] As a consequence, the remainder of the curves were all calculated using the modified strip method with 200 spanwise strips. A further interesting feature of the curves in Fig. 9a is that the rolling moment decreases steadily as the span ratio increases. Such is not the case when the second vortex of the pair is included in the computations (Fig. 9b). One important consequence of the second vortex in the pair is that the maximum rolling moment may be larger than it is for an isolated vortex. Whereas the rolling moments on small following aircraft are about the same, substantial differences occur when the span ratio is about 0.8 or larger. Furthermore, the largest rolling moment does not occur when the following wing is aligned with a vortex center as is the case when the span ratio is small (Fig. 10a). That is, as the wing tip of the following wing approaches the other vortex in the pair, the rolling moment can be enhanced by more than a small amount. Hence, when flight tests are conducted and penetrations are made into vortex cores marked by smoke, it should be borne in mind that larger rolling moments may be present elsewhere; for example, somewhere between the two vortices when the span ratio is about 1.0 (Fig. 10b). At larger span ratios of about 2.0 (Fig. 10c), the location of maximum torque in a vortex pair occurs when the center of the following wing is outboard of the centers of the vortices.

Prospects for Adequate Alleviation

Consideration is given here to the various wake-alleviation possibilities that have been tried in order to identify those that

a) Initial configuration of wake vortices as marked by oil mist smoke generators

b) About one mile behind aircraft

Fig. 11 Photographs of wake configuration shed by B-747 with its inboard flaps fully deflected, with outboard flaps and landing gear stowed.[60,61]

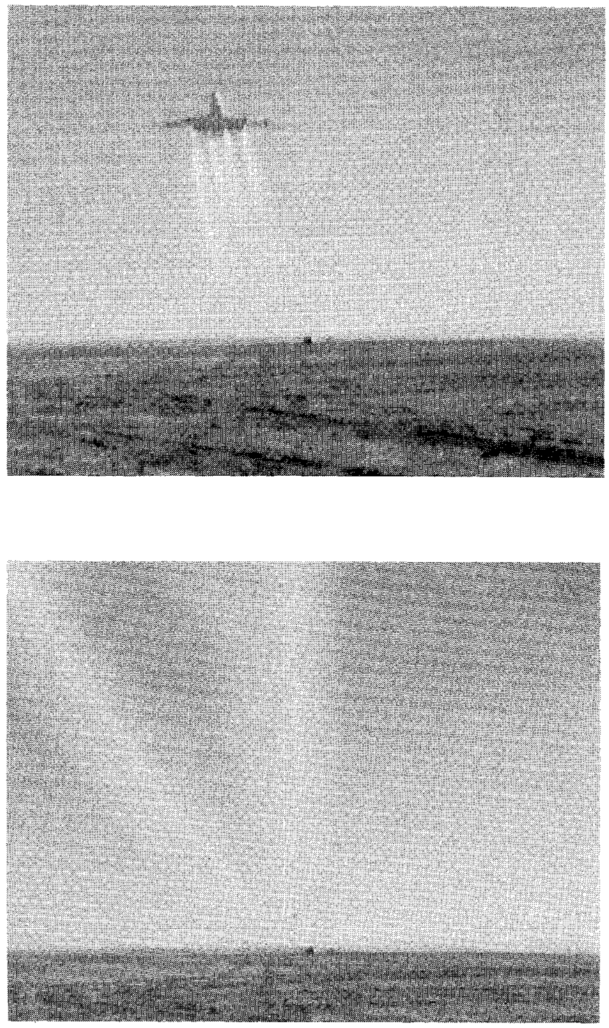


Fig. 12 Photographs of wake configuration shed by B-747 with its inboard flaps fully deflected, with outboard flaps stowed and landing gear extended.[60,61]

may yield the fourfold or more reduction in the rotary velocities needed to make the wakes of present large aircraft tolerable. One of the first mechanisms observed to bring about the disruption and dispersion of wake vortices was a sinusoidal self-induced instability identified by Scorer[55] and correctly explained by Crow.[56] The instability, which is now regularly observed to occur behind high-flying jet aircraft, produces regularly spaced waves on the vortices that lead to the linking of the vortices across the span, so that the vortex pair is converted from two nearly straight lines into a sequence of irregularly shaped loops of vorticity. The loops of vorticity spread the vortical region behind the aircraft and enhance the decay and dispersion of the wake velocities. Although the alleviation process is effective, the time required to accomplish the needed wake spreading is unacceptably long and, under certain conditions, the process does not occur at all. Therefore, efforts have been made to insure and accelerate the onset[57,73–76] of the Scorer-Crow instability by manipulating control surfaces (or the entire aircraft) in order to impress waves on the vortex lines as they are generated to reduce the time required to achieve rapid growth in wave amplitude.

Extension of the sinusoidal-wave and loop-forming instability to wakes consisting of multiple vortex pairs has occurred on several occasions. The first published occurrence[60–64] was observed in the wake of the B-747 transport aircraft when it had only its inboard flap deployed to produce a wake consisting of three vortex pairs. The loop formation that occurred between the two inboard vortices on each side of the fuselage (Figs. 11a and 11b) was damped by the turbulence shed by the landing gear or by a slight yaw of the aircraft (Fig. 12). As a result, the alleviation occurred only under very limited circumstances. In another attempt to find a span loading that was effective in alleviating wake velocities, the outboard flaps of the B-747 were deployed instead of the inboard flaps. It was found that there was no alleviation and that the wake intensity was actually increased by such a change in the span loading. These results demonstrate the sensitivity of the loop-forming instability to turbulence (vortex core size) and to the arrangement of the vortices in the wake.

The effectiveness of giving an initial displacement to wake vortices to trigger or enhance the likelihood of wave growth of an instability for loop formation was demonstrated in flight with a B-747 aircraft. As mentioned previously, during the latter part of a test program[71,72] to evaluate the effectiveness of spoilers for alleviating wake velocities, the B-747 was given a series of roll oscillations to find out if maneuvers were detrimental to the wake alleviation achieved with spoilers. To everyone's surprise, the wake became even less hazardous, so that the pilots of the probe aircraft described the ride through the wake as resembling a ride over a rough road, without a coherent rolling moment. When the same experiment was performed with an L-1011, no change in wake alleviation was achieved. Subsequent analysis[74] indicated that the span loading on the B-747 approached a three-pair configuration closely enough when spoilers were deployed to enable the roll oscillations to trigger or initiate a loop-forming instability between the two inboard vortex pairs. The maneuver required for such an alleviation mechanism is, of course, impractical, but an understanding of the concepts may help in the search for a useful and effective solution. Although the configurations discovered in these tests have serious drawbacks, they demonstrate that alleviation can be achieved, and they have done much to promote research and understanding of wake-alleviation mechanisms.

In an effort to find vortex wakes that disperse rapidly, span loadings that shed a number of vortex pairs were studied theoretically[58,65] and experimentally.[50,60,61,64] It was found that when the vortex pairs are of comparable strength, they interact to bring about large excursions and loop formation between vortices so that the wake vorticity is quickly dispersed. However, it was also found that when the angle of attack of the wing is increased to the point where the wing-tip vortex becomes dominant, the wake organization is restored so that alleviation is diminished. Therefore, there remains a need to develop guidelines for the design of lifting surfaces that produce highly dispersive multiple vortex wakes. The penalties associated with these designs can then be evaluated to reveal whether implementation of such an alleviation scheme is feasible.

An alleviation mechanism that was tried early in the program was turbulence injection[48,50,59,61,70–72] by use of spoilers or other surfaces oriented nearly perpendicularly to the oncoming airstream. Considerable effort was expended in this area because it was likely that the spoilers already onboard most large transport aircraft might be sufficient as turbulence generators for the alleviation needed. Hence, the cost of implementation would be quite reasonable. The alleviation achieved with spoilers consists primarily of a reduction in the rotary velocities near the core of the vortices.[48,50,59,72] None of the configurations tried produced enough alleviation to satisfy the requirements of a 2-n.mi. spacing. As far as future prospects are concerned, further increases in alleviation are not likely because it can be reasoned that the size of the mixing eddies needed to neutralize the wake vortices should be on the order of the wingspan. The eddies produced by onboard turbulence devices are probably less than $0.1b_g$, which is too small to demolish the vortex-wake structure.

As already mentioned, several configurations have demonstrated theoretically and experimentally that vortex interactions are an effective means of bringing about the large-scale mixing needed for wake alleviation. It was proposed, therefore, that vortices be injected into the wake to enhance the process.[66,68] The devices used were vertical surfaces on the wing set at angle of attack to the local airstream. The measured reductions in the wake-induced rolling moments were appreciable and adequate for the 2-n.mi. separation distance if the results for the wind-tunnel models are valid at flight scale. Since the alleviation mechanism and, therefore, the theoretical guidelines for the design are not known in detail, implementation of vortex injection by means of wing fins or some other device is not straightforward at this time. Therefore, vortex injection appears to be a promising scheme that needs further study before the alleviation mechanism involved can be clearly defined and guidelines for implementation and optimum application delineated.

Other concepts not mentioned here may result in an alleviation mechanism that will produce the alleviation needed for a 2-n.mi. spacing. Whether any of the concepts will ever be of such a form that they can be retrofitted onto the existing fleet of transport aircraft is questionable, but this should not be used as a deterrent to exploration of alleviation schemes. One quite severe requirement is that the alleviation be effective under all weather conditions and that it not be susceptible to atmospheric structure or proximity to the ground.[76]

Concluding Remarks

A review of the literature on wake-vortex encounters in flight and in flight simulators has provided an estimate of the level to which the vortex-induced rolling moments must be reduced in order to be perceived as nonhazardous at a 2-n.mi. separation distance. As expected, the criteria are based on the ratio of the vortex-induced acceleration in roll to the aileron-induced roll acceleration. This ratio is the same as the ratio of the static rolling moments induced by a vortex to that induced by aileron deflection. The several cases measured in flight tests and the results of ground-based simulations of vortex encounters indicate that a wake is acceptably alleviated if the ratio of vortex-to-aileron rolling moments is less than about 0.5. Based on the roll-control capability typical of transport aircraft, the wake-vortex-induced rolling-moment coefficients must be less than about $C_{l_v} = 0.03$. Since vortex-induced rolling-moment coefficients are on the order of $C_{l_v} = 0.12$ behind large wide-body transports, the rotary velocities in vortex wakes must be reduced by a factor of about 4. Consideration of wake-alleviation schemes explored in the past indicates that such a reduction will be difficult to achieve when other constraints of aircraft performance are included, but the problem solution does not appear impossible. An increase in the onboard automatic control capability of the penetrating aircraft eases the wake-alleviation requirements somewhat but does not change the need for a substantial reduction in wake rotary velocities.

When a satisfactory alleviation scheme has been identified, the alleviated vortex structure should be inserted into a simulator to determine whether the maximum bank angles induced are within tolerable limits and whether a variety of pilots feel that the wake is safely tolerable to any aircraft that may encounter it. This added requirement should be fulfilled before consideration is given to implementation. Such a step is necessary because the structure of the alleviated wakes will no doubt be drastically different from the Lamb vortices used in previous simulator studies.

Appendix: Methods for Force and Moment Computations

A precise calculation of the flowfield around an aircraft as it encounters the velocity field of a lift-generated wake requires a complete knowledge of the structure of the three-dimensional time-dependent wake, of the control inputs by the pilot, and of the motion of the aircraft. Presumably, the aircraft motion could be calculated if the aerodynamic loading or pressure distribution on the surface of the aircraft were known. Since a well-defined wake structure is difficult to obtain, many of the studies of vortex interactions were carried out with stick and flat-plate models of the penetrating aircraft. The aerodynamic loads were estimated by the so-called strip theory, in which local forces are based on the angle of the flat-plate surface relative to the local wind and a two-dimensional lift-curve slope. Since such a method usually overestimated the aerodynamic forces, various corrections were made to the lift-curve slope to account for the finite span of the wing and tail surfaces. One of the first was simply to assume that the surface loading is approximately elliptical. Unfortunately, the loading is more often split into positive and negative areas on the two sides of the station where the vortex crosses the lifting surface. This characteristic was utilized by Munk[36] in the analysis of the span loading on a rolling aircraft. He observed that a good first approximation to such a situation is obtained by assuming that the loading on the two halves of the wing is approximately elliptical. That observation calls attention to the fact that any change in the sign of the angle of attack can be treated as if the wingspan is segmented at that spanwise station and loaded elliptically. Somewhat later, Maskew[41] suggested that not only should all of the lifting surfaces be treated in this way but also, when the strip theory is used to calculate the force distribution, the lift-curve slope should be adjusted from the two-dimensional value by a formula developed by R. T. Jones (Ref. 38, p. 95), which is based on the planform of the wing and on the nodes of the upwash.

Each of the foregoing strip-theory-type calculation methods assume that the oncoming stream is uniform in magnitude and direction. The angle-of-attack variation brought about by the vortex structure is then assumed to be accomplished by twist in the wing. When consideration is given to the boundary conditions to be specified on the panels, it becomes apparent that the geometry can be thought of in two ways. In the first, it is assumed that the wing is in the $z = 0$ plane and that the local angle-of-attack values are taken from the vortex wake. Although such a model approximates the real flowfield, it introduces vorticity into the flowfield that is assumed to be irrotational in the potential flow solution. In the second, it is assumed that the oncoming stream is uniform in velocity so that it is irrotational but that the wing is twisted along its span so that the local angle of attack corresponds to the up- and downwash imposed by the vortex wake. Both models have conceptual disadvantages, but the predicted loads are the same to the degree of accuracy of the theories.

The formalism of the problem was clarified somewhat when Barrows[43,45] called attention to a reciprocal theorem developed by Heaslet and Spreiter,[37] which relates an arbitrary downwash field to that on a fictitious wing in steady rolling motion. Barrows discusses the relationship further and summarizes some calculations that used the theorem for similar studies. In brief, the theorem formalizes and justifies the assumption, which is usually made, that the loading on the encountering wing can be calculated by using various theories that treat the oncoming stream as uniform and the vortex downwash field as wing twist (or as local angle of attack). The magnitude of the error in such an approximation is probably not large when the flow angularities are small. However, when the angles of attack are large (as they often are near the center of the vortex) and when the energy or momentum of the impacting fluid is limited to the annulus of fluid rotating in the vortex, the pressure field would seem to be susceptible to considerable error. In the absence of a better approximation and one that is as convenient to use, the reciprocal process will be assumed to apply with an acceptable accuracy. Other sources of inaccuracies, e.g., viscous losses, are also assumed to be negligible. These approximations were found to be valid approximations in a series of experiments[50] in a wind tunnel to the accuracy of the measurements. It was also found that the method suggested by Maskew provides good or acceptably accurate results.

The next step up in method sophistication is the vortex-lattice method,[41,42] in which the camber line of the encountering wing is approximated by vortex panels located in the $z = 0$ plane, that is, in the horizontal plane. Furthermore, the method seems to provide quite good results.[50]

More sophisticated representation of the aircraft surfaces with panels located at the true surface positions would provide a better representation of the penetrating aircraft but still leaves questions as to how to represent the swirling flowfield of the vortex wake. The most complete analysis would be to carry out a finite-difference solution for the flowfield using a Navier-Stokes formulation that accounts for the changes in direction and in total head of the airstream in the wake that is impinging on the encountering aircraft. The added complexity needed for such a solution would be of value, if only to find out whether the simpler methods contain significant errors in concept or in the estimated magnitudes of forces. Use of a Navier-Stokes solution technique for estimating a dynamic interaction would be too cumbersome and time-consuming for any anticipated needs at the present time. A large number of parametric variations for conceptual studies are also best done with the simpler methods, with care being taken to realize the approximations made and the possibility for misleading results.

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